

Applying the boundary conditions in the first and second equations of (12), one obtains

$$A_1 = - \frac{Qa^2}{G_0\alpha} \frac{J_2(\alpha a) e^{(n_2-n_1)b}}{n_1 e^{(n_2-n_1)b} - n_2} \quad (14)$$

and

$$B_1 = -A_1 e^{(n_1-n_2)b}$$

Then the displacement  $(u_\theta)_{z=0}$  is given by

$$(u_\theta)_{z=0} = \frac{Qa^2}{G_0} \int_0^\infty \left[ \frac{e^{(n_2-n_1)b} - 1}{n_1 e^{(n_2-n_1)b} - n_2} \right] J_2(\alpha a) J_1(\alpha r) d\alpha \quad (15)$$

To get an idea of how the displacement  $u_\theta$  changes on  $z = 0$  for different values of  $r$ , take  $m = b = 1$  and  $a = k_0 = 0.5$ . The result is given in Table 1.

### Similar Solutions in Boundary Layer Slip Flow

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SOME methods of solution for the boundary layer slip-flow problem have been suggested (see, e.g., Refs. 1-3). To check the accuracy of these solutions, one needs to compare them with exact solutions. The purpose of this note is to show how an exact solution can be obtained by means of the similar-solutions technique.

The basic equation of the incompressible boundary layer in terms of Von Mises' nondimensional coordinates can be written as

$$uu_x = U_e U_{e,x} + u(uu_y)_y \quad (1)$$

where  $U_e$  is the outer velocity, with the following boundary conditions:

$$u_y(x,0) = k \quad (2)$$

$$u(x,\infty) = U_e \quad (3)$$

$$u(0,y) = g(y) \quad (4)$$

Now put  $u^2 = f(x) F(z)$ , with  $z = y/h(x)$ .

The functions  $f(x)$  and  $h(x)$  must be found from Eqs. (1) and (2); one obtains

$$f^{1/2} = h = U_e = 1 + Cx$$

To obtain the function  $F(z)$ , one needs to solve the following equation:

$$F - (zF'/2) = 1 + (F^{1/2}F''/2C) \quad (5)$$

with the boundary conditions

$$F(\infty) = 1 \quad F'(0) = 2kF^{1/2}(0)$$

Observe that the function  $g(y)$  of Eq. (4) is given by  $F(y)$ . Equation (5) has been solved numerically, and the results are shown in Fig. 1 for  $C = 0.5$  and  $k = 1$ . Now a satisfactory approximate solution is given which is obtained by substituting for  $F^{1/2}$  in Eq. (5) a mean value  $F_m^{1/2}$  ( $0 < F_m < 1$ ). One has then

$$F = 1 + A(1 + 2\zeta)(1 - (\pi\zeta)^{-1/2} \exp(-\zeta) \times \{ {}_1F_1(1, \frac{1}{2}, \zeta) - [1/(1 + 2\zeta)] \})$$

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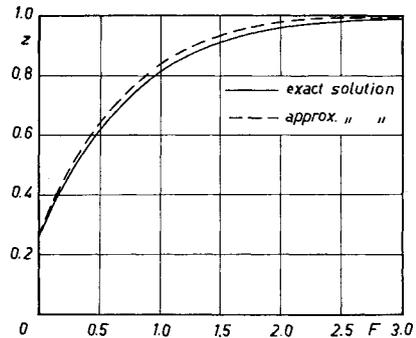


Fig. 1 Exact and approximate velocity function

where  ${}_1F_1$  is the confluent hypergeometric function,  $\zeta = Cz^2/2F_m^{1/2}$ , and the constant  $A$  is given by

$$A = \frac{\pi k^2 F_m^{1/2}}{4C} - \frac{\pi k}{4} \left( \frac{k^2 F_m}{C^2} + \frac{8F_m^{1/2}}{\pi C} \right)^{1/2}$$

In Fig. 1 is shown this approximate function obtained by assuming for  $F_m$  the value 0.5.

#### References

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- <sup>2</sup> Hassan, H. A., "On skin friction in the slip-flow regime," *J. Aerospace Sci.* **28**, 335-336 (1961).
- <sup>3</sup> Pozzi, A. and Renno, P., "Slip flow with axial pressure gradient," *J. Aerospace Sci.* **29**, 1393 (1962).

### Stress Concentrations around a Small Rigid Spheroidal Inclusion on the Axis of a Transversely Isotropic Cylinder under Torsion

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#### Introduction

IN this note, stresses due to a small rigid inclusion in the form of an oblate spheroid situated on the axis of a large transversely isotropic cylinder under torsion have been found. A corresponding problem for a spherical inclusion in a similar medium was considered by Chatterji.<sup>2</sup> From the results obtained here, stresses due to a rigid inclusion in the form of a prolate spheroid can be deduced by suitable modification.

#### Solution

The strain-energy function of a transversely isotropic material in cylindrical coordinates is given by

$$W = \frac{1}{2}c_{11}(e_{rr}^2 + e_{\theta\theta}^2) + \frac{1}{2}c_{33}e_{zz}^2 + c_{13}(e_{rr} + e_{\theta\theta})e_{zz} + c_{12}e_{rr}e_{\theta\theta} + \frac{1}{2}c_{44}(e_{\theta z}^2 + e_{rz}^2) + \frac{1}{2}c_{66}e_{r\theta}^2$$

where

$$c_{12} = c_{11} - 2c_{66}$$

Considering the large twisted cylinder under torsional stresses

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